The Use of GARCH Autoregressive Models in Estimating and Forecasting the Crude Oil Volatility

Radu-Cristian MUȘETESCU
Bucharest University of Economic Studies, Romania
radu.musetescu@rei.ase.ro

George-Eduard GRIGORE
Bucharest University of Economic Studies, Romania
grigoregeorge15@yahoo.com

Simona NICOLAE
University ”Politehnica” of Bucharest, Romania
nsim2005@yahoo.com

Abstract
Today, oil is one of the most popular commodities traded globally, due to its indispensable character and multiple properties offered to mankind. Increased attention is paid to the analysis of volatile and fluctuating trends in the overall price of this valuable energy source. Using the autoregressive conditional heteroskedasticity models such as GARCH(1,1), GARCH-M(1,1) and EGARCH(1,1), the present study has as a priority objective in estimating and predicting the volatility of the oil returns series (Brent Crude Oil return series) in the 1987-2022. The main results highlighted the preference in using the asymmetric model EGARCH (1,1) on the measurement of conditional variance, showing that Brent Crude Oil reacts over 90% to any existing market's shock (i.e.: information, events, facts, news, etc.) in a negative manner/way. At the same time, various tests and evaluation conditions were used (ARCH-LM Test, Durbin-Waston Test, High Log likelihood, Lowest Schwarz Information Criteria) in investigating the level of performance in estimation the conditional crude oil volatility. Each GARCH (1,1) model is meeting brilliantly these conditions and acquiring the character of stability and validity in use. At the same time, performing forecast analysis on crude oil volatility in two different time periods: 1987-2022, respectively 2020-2022, it was shown that existence of the phenomenon of clustering-volatility over the time, with strong implications for the functioning mechanism of international financial markets. Fulfilling those restrictive conditions, the symmetric and parametric model GARCH-M (1,1) becomes, in our case, the most efficient model in forecasting the volatility of Brent Crude Oil return series in the analysed period.

Keywords: Conditional variance; GARCH models; crude oil returns; clustering-volatility; COVID-19 Pandemic;

JEL Classification: C10; C50; C53; C59; Q43

DOI: http://doi.org/10.24818/ejis.2022.02

1. Introduction

Increasingly fluctuating global oil prices and the impact on the transactional, decision-making and investment mechanism in global markets are the main and relevant concerns found in the literature. From this point of view, many studies focus on the study of crude

[1] Received: 12 March 2022; Revised: 7 April 2022; Accepted: 15 May 2022

oil volatility and how it has multiple implications for macroeconomic indicators in a country (Sekati *et al.*, 2020; Yildirim, 2017; Hu *et al.*, 2020).

It should also be noted that the crude oil is a vital commodity of modern economic and it can be used as an important source of energy or as a source of raw materials (Sekati *et al.*, 2020; Yi *et al.*, 2021; Dunn and Holloway, 2012) and this is the most important, relevant and widely traded commodities that effects the global economy and international trade (Wakeford, 2008). Starting from these specific features regarding the place and the extremely important role of oil on the international paradigm, our study aims to estimate and forecasting the conditional volatility equation in oil return series. Therefore, the quantitative analysis had as a starting point the determination of the daily oil returns series in the period 1987-2022, data obtained from the U.S. Energy Information Administration (EIA) database.

The objective of the study can be justified by the current uncertain and unpredictable period that we are crossing it, and which can be explained by the main effects produced by the COVID-19 pandemic. This is also empirically demonstrated by the fluctuating movements and frequent shocks that affect the dynamics of the global crude oil price (in our case, Brent Crude Oil return series). Thus, we are convinced that by estimating and providing predictions on the future oil returns and volatility is essential for determining/calculating asset prices, hedging, derivative pricing and these provide valuable information to players in the market about the uncertainty.

From a methodological point of view, following the statistical analysis of the oil return series, we will apply ARCH-GARCH autoregressive models to estimate as accurately as possible the conditional variance of daily oil returns. Compared to other studies (Yildirim, 2017; Haque and Shaik, 2021) that used West Texas Intermediate (WTI) that serves as a reference price for buyers and sellers of crude oil, our study chooses the Brent Crude as the benchmark that is primarily used in Europe/European area.

We opt for the use of three such models, two of which are symmetrical and linear models (GARCH and GARCH-M), respectively the asymmetric and non-parametric EGARCH model. These models have the ability to calibrate time-varying conditional volatility and incorporate the past observations into the future volatility, being extremely popular in the recent literature (Narayan and Narayan, 2007; Kulikova and Taylor, 2013; Wang *et al.*, 2016; Yildirim, 2017; Charles and Darne, 2017; Oyuna and Yaobin, 2021). Also, these extension models seek out to improve the GARCH model to capture the main characteristics of the oil return series.

In our study, being aware of the important role of measuring the level of volatility oil returns, we tested and verified each autoregressive model applied, so that we can compare their performance and usefulness. To choose the most fitted model, we will apply relevant statistical tests verifying the statistically significance of the obtained parameters, implicitly performing the residual tests on fulfilling the conditions of no heteroskedasticity and no serial correlation in residual terms series. Moreover, the study aims to test the ability of the model (s) to predict developments using the mentioned period as a database - analysis of the crude oil return series conditional volatility in two different time periods: 1987-2022 and respectively 2020-2022. As for improved GARCH models, this study employs three error measures to evaluate the forecasting performance of models: Mean Absolute Error (MAE); Mean Absolute Percent Error (MAPE) and Root Mean Square Error (RMSE).

In a comparative approach, we can see which autoregressive model is performing in estimating the conditional variance, respectively which is effective and accurate in predicting volatility in these analysed time intervals. At the same time, the information obtained from the use of GARCH (1,1), GACH-M (1,1) and EGARCH (1,1) can explain the persistence of the crude oil volatility in the near future, but also how the global crude oil price reacts and is sensitive to any shock, with major implications for global market participants.

The paper is organized in the following order: section 2 presents the most relevant literature review; section 3 describes the Research Methodology, Data, Preliminary Analysis of oil return series and the applied GARCH Models (GARCH, GARCH-M and EGARCH); section 4 discusses the main results and interpretation of these results and section 5 presents the conclusion.

2. Literature Review

This section illustrates the review of major research aimed at studying and investigating the degree of volatility in the case of oil returns. From this perspective, the literature is increasingly concerned with reporting and managing the increasing influence, but also with the multiple effects (positive or negative) of this valuable resource, crude oil. In this sense, a central place in the literature is occupied by the estimation of the degree of volatility of oil returns and finding appropriate methods to provide a high degree of forecasting and predicting this volatility/variance in the near future. The relationships and the interdependencies of an economic, political, social or geopolitical nature that they have has at the macroeconomic level represents the fundamental landmarks analysed by Yi *et al*. (2021).

The study conducted by them provides relevant explanations on how it works and improvement of the crude oil futures market in China (China INE crude oil futures market). Using GARCH-MIDAS autoregressive models, Yi *et al*. (2021) main purpose is to study how different factors (i.e., geopolitical risk indices, economic policy uncertainty and infectious disease pandemic indices) are used to measure uncertainty on market, can be explained by determining the conditional variance of return oil series. Statistically, the results obtained showed that episodes of uncertainty more and more widespread in the case of the analysed market continues to persist, especially in the recent period, characterized by Covid-19 pandemic. Also, by introducing these uncertainty indicators into the GARCH-MIDAS, it has been shown that the volatile trend in the case of INE Crude Oil Futures Market has an extremely high impact on economic activity as a whole and the implementation of macro-prudential policies in risk management is increasingly needed.

Currently, most global crude oil markets are experiencing an extreme variety unusual events and situations, which may necessitate the formulation and application of concrete actions to achieve a satisfactory degree of stability (i.e., formation of crude oil benchmark prices in Asia). In this regard, other studies by Yi *et al*. (2021), Sheng *et al.* (2020), Brandt and Gao (2019), aimed at establishing a regulated trading framework for crude oil in the Southeast Asia, given that these countries are net importers and consumers of energy resources (Zhang and Ma, 2021; Wang and Wu, 2012; Zhang *et al.*, 2015). Thus, these studies illustrate the measurement of global oil price volatility as well and the ability of the countries in this region to protect themselves from the current and frequent situations of fluctuation in crude oil prices.

The predilection for intensifying the use of specific risk management tools is found in research by Hu *et al.* (2020); Miao *et al.* (2017), according to which measuring oil price volatility is becoming a priority in the uncertain current global context. At the same time, these studies propose asymmetric autoregressive models (i.e., EGARCH, FIGARCH and TGARCH), so that the increased impact of any type of information in predicting global oil price volatility. Other studies (Escribano and Valdes, 2017; Brandt and Gao, 2019; Wei *et al.*, 2017) show that crude oil it is also a tool for exercising political power globally, being one of the most often a trigger for conflict events and situations that increase the political and geopolitical risk. From this point of view, crude oil becomes a geopolitical weapon, which can often lead to multiple situations of panic and fear among the investors and the participants in the international financial market. Thus, Yi *et al.* (2021); Zhang and Wang (2015); Wei *et al.* (2017), suggest that the higher the price of oil, the more volatile it is persistent events with a negative impact on the international macroeconomic framework.

At the same time, more recent research (Bai *et al.*, 2020; Sheng *et al.*, 2020) has shown that since the Covid-19 pandemic, which changed that economic order and global policy, more attention is paid to the degree of risk estimation and forecasting specific to crude oil returns, but also finding appropriate rebalancing solutions balance of market participants and government policy makers. The unpredictable events of the current pandemic crisis have led to many distortions and shocks to global oil markets; with increasing consequences frequent fluctuations in the benchmark price of this highly traded asset (Baker *et al.*, 2020; Demirer *et al.*, 2018).

For these reasons, the autoregressive GARCH models on estimation conditional variance of oil return series are increasingly and commonly used in scientific and empirical literature (Bollerslev, 1986; Engle and Bollerslev, 1986; Pagan and Schwert, 1990; Nelson, 1991; Bera and Higgins, 1993). We draw attention to the study conducted by Liu *et al.* (2019), where through the non- GARCH-MIDAS parametric short-term weighted volatility was investigated series of daily oil returns between 1986-2018, and introducing specific indices on measuring geopolitical risk (GPR and GPRS) suggested high correlation with the oil return series. Similarly, Zang and Ma (2020) showed that GARCH models are more efficient by incorporation of uncertainty factors, providing the most realistic estimates of the conditional variance for oil return series. Geopolitical risk is an important factor that negatively influences the future of the market oil futures contracts in China, a trend under the impact of uncertainty from the UK or Japan (Yi *et al.*, 2021).

On the other hand, in the literature is highlighted the increased importance in use ARCH-GARCH autoregressive models to model macroeconomic and aggregate variables. In this regard, the study by Sekati *et al*. (2020) is positioned accordingly whose impact was analysed by applying the ARCH, GARCH and EGARCH models macroeconomic variables on the global price of crude oil for South Africa. The variables used were GDP, inflation rate, exchange rates and the global oil price (WTI). Carrying out the analysis in the period 1990-2018, the results suggested that at the time which is estimated to increase by 1% for each variable, the price of oil reacts in the same meaning, but the growth rate being different.

Also in the same article, the parameters obtained by applying the GARCH (1,1) and ARCH (1) were statistically significant, indicating the negative impact of the rate inflation, as well as the positive influence of exchange rates and GDP on crude oil prices. According to the results from the use of EGARCH (1,1), the global price of crude oil is adversely affected by each macroeconomic indicator, suggesting the need the application of economic actions and policies that will be able to temper the increased volatility in the long run in South Africa. It can be confirmed that the volatile movements of the global oil price led to a series of problems with a considerable impact on the performance of the economic activity of the analysed state (Sekati *et al.*, 2020).

More and more researchers have come to the conclusion that the persistence of the global price has a negative impact on the standard of living in most poor or poor countries development course (Yildirim, 2017; Demirer *et al.*, 2018); others affirming / validating the unstable character of crude oil prices compared to other non-financial assets (Adelman, 2000; Lipsky, 2009). Last but not least, the study of oil return's volatility is becoming a topic that illustrating the multiple or possible effects within each field of activity: economic, political, social and financial sector.

In other words, considering the fact that crude oil is one of the most important sources of energy, but also the most widely traded resource globally (Guo and Kilesen, 2005; Dunn and Holloway, 2012; Wakeford, 2008), the more it is necessary to study the multiple interdependencies between macroeconomic indicators and their impact on the global price of crude oil. This can be achieved by using autoregressive estimation models measuring volatility at different time periods. From this point of view, the ARCH-GARCH models initiated by Bollerslev (1986), Engle and Bollerslev (1986) became more and more common for investigation the influence of the global price of oil on the dynamic and evolutionary mechanism of global markets. For example, Jin (2008) highlighted in the comparative manner/approach that the upward trend in oil prices has an impact positive growth rate in the countries under analysis (Federation Russia, China and Japan). The symmetric and asymmetric GARCH models used in the study by Wei *et al.* (2010) led to the most accurate estimates of the long-term volatility of crude oil return series.

Global oil price shocks show significant negative effects on the exchange rate in the South Africa, which following the application of the EGARCH exponential model (1,1), in the study developed by Kutu and Ngalawa (2017). Other combinations of the autoregressive models (i.e., GARCH-M, EGARCH, TGARCH, IGARCH, etc.) had as a central result the estimation of oil price volatility and the identification of a high degree of persistence in the near future (over 80%). These aspects are illustrated in testing the validity and stability of autoregressive models into the oil return series analysed by Agnolucci (2009); Ramzan *et al.* (2012).

In line with the objective of estimating and providing valuable predictions on the oil return series is positioned the study conducted by Zhang *et al*. (2019). The results of this study had two implications: (i) evaluating the performance of the MRSGARCH (two-regime Markov Switching Regime GARCH) compared to the resulting estimates from the use of traditional GARCH models; (ii) extension of the stochastic model of MRS-GARCH volatility in determining the Value-at-Risk (VaR) function. At the same time, other studies have focused on the analysis of MRS-GARCH models (Zhang *et al.*, 2019; Manera *et al.*, 2007), emphasizing the novelty in identifying the episodes of transition on measuring the volatility of oil return series in different time intervals/periods.

Crude oil volatility is a central element that is often extended to VaR and Conditional VaR models, therefore, can provide the highest possible level of estimation (Zhang *et al.*, 2019; Zhang and Wang, 2015). Increasingly use of stochastic volatility models to the detriment of models GARCH becomes a point of interest in the study conducted by

Oyuna and Yaobin, (2021) in forecasting volatility crude oil returns. The innovative manner of this study is the launch of the Heston model to identify detection of Jump effects by the Euler-Maruyama Scheme mathematical simulation. Another contribution of this study was the understanding of the behaviour of oil prices and the way in which volatility persists over non-oil-producing countries, respectively oil-producing countries.

From the perspective of the participants in the international financial markets, the importance of forecasting the future variance of crude oil is growing in today's unpredictable conditions and uncertain, emphasizing the impact of the application of risk protection measures (Charles and Darne, 2017; Oyuna and Yaobin, 2021). It follows that GARCH models can capture a number of relevant factors (the stylized facts: volatility clustering, asymmetry extended memory volatility, heavy-tailed distributions or fat-tail property of financial data) to the oil return series, highlighted by Wang *et al.* (2016); Kulikova and Taylor (2013).

Comparative assessment of autoregressive methods of estimating and predicting daily volatility of crude oil return series (i.e.: RiskMetrics, GARCH, IGARCH, Exponential GARCH and Markov-Switching GARCH) is the main objective of the study by Herrera *et al.* (2018). Regarding the analysed period, between 2007-2015, the authors state that the global crude oil price crossed many controversial and unpredictable moments, confirming that: "*a time period that comprises the rapid growth in oil production, the large upswing in oil prices during the economic expansion of the early 2000s, the downswing following the 2008-2009 global financial crisis, and the sharp decline since the second semester of 2014*" (Herrera *et al*., 2018, p.5). The obtained results showed that the GARCH model (1,1) offers increased predictions a short-term volatility, while the EGARCH (1,1) and MRS-GARCH (1,1) models are suitable in modelling/estimating the medium- and long-term volatility.

The negative effects of the current pandemic crisis on the crude oil prices are investigated by Haque and Shaik (2021). Using autoregressive models established in empirical research such as Autoregressive Integrated Moving Average (ARIMA) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH), the authors launched a quantitative research on estimating the volatility of WTI oil series from 10 February to 27 April 2020. The results of the study stated that ARIMA models can provide high-level confidence in predicting volatility in increasingly extreme situations. In the same study, it was shown that the fluctuating trend of the global oil price has increased effects on the entire economic activity of a state, and especially in key economic sectors: manufacturing, transportation. In a comparative manner, we will estimate and forecast the crude oil volatility specific to the oil return series in the longer period: 24 January 2020 to 24 January 2022.

In a similar approach, Yaziz *et al.* (2011) used comparatively ARIMA and GARCH models for the forecasting the crude oil volatility in the financial post-crisis periods. The information obtained showed that the GARCH model (1,1) is increasingly performing in calibration the conditional variance equation compared to using the ARIMA model (1,2,1). Other authors have pointed out different points of view on the advantages offered by the autoregressive models (Ahmed and Shabri, 2014; Chen *et al.*, 2014; Neshat *et al.*, 2018).

The intense investors' concern in setting future measurement models in estimating and forecasting the return and volatility series (especially crude oil, natural gas, gold) is the basic hypothesis specified in the research conducted by Yildirim (2017). Specifically, he applied the ARCH-GARCH models for estimating the conditional variance equation on the crude oil series in 2015-2016. The results show that crude oil remains a strategic and often traded asset, but with a high level of volatility of over 75%, measured by the sum of the parameters ARCH and GARCH. At the same time, GARCH (1,1) remains one of the most recommended and fitted models in the world being able to identify the volatility for non-financial, financial assets or stock indices. Similar results were reported by the study conducted by Er and Fidan (2013).

The central objective of studying the phenomenon of persistent crude oil volatility by using asymmetric and symmetric autoregressive models is suggested by the effort of Mohammadi and Su (2010). Characterizing the behaviour of weekly return series in the period 1997-2009, at the level the 11 international/global markets, it was shown that the volatility is persistent over the time and also the presence of ACH effects is larger (i.e. volatility-clustering and jumps period). Also, "*there are some indications that the conditional standard deviation is better able to capture the volatility in oil returns than the traditional conditional variance*" (Mohammadi and Su, 2010, p.107). Furthermore, the study by Saltik *et al.* (2016) states that the asymmetric models (EGARCH, FGARCH and FIAPARCH) are fitted and accurate in predicting the future conditional variance at the level of global crude oil price (WTI) and natural gas (Henry Hub) in 2009-2014 and 2010-2014.

3. Research Methodology

Given the estimation and forecasting of volatility oil return series, our study addresses different autoregressive models and methods (ARCH-GARCH type) used in many specialized studies (Mohammadi and Su, 2010; Saltik *et al.*, 2016; Haque and Shaik, 2021; Yildirim, 2017; Yi *et al.*, 2021). Assuming that oil is a global resource with an increased impact on macroeconomic indicators and whose high influence in the macroeconomic level (Sekati *et al.*, 2020; Brandt and Gao, 2019; Zhang and Wang, 2015) of increasing importance, it is necessary to analyse and investigate the high degree of volatility and measure persistence at different times of Brent Crude Oil return series.

From this perspective, our quantitative approach highlights the descriptive and statistical analysis of the Brent Crude Oil return series, respectively, the estimating and providing predictions of the risk associated with these returns by applying the ARCH-GARCH conditional autoregressive methods. Our study proposes a comparative approach in estimating the crude oil volatility specific to the daily oil return series in 1987-2022. It is also necessary to forecast this conditional variance in the last two years, considering the increasingly fluctuating movements in the global crude oil price. In this regard, the GARCH (1,1); GARCH-M (1,1) and EGARCH (1, 1) were used. The first two models are part of traditional and symmetrical (linear) calibration models and the EGARCH model is an asymmetric and nonlinear model, with the ability to incorporate the impact of information in measuring this persistence of volatility.

3.1. Data analysis and investigation the oil return series

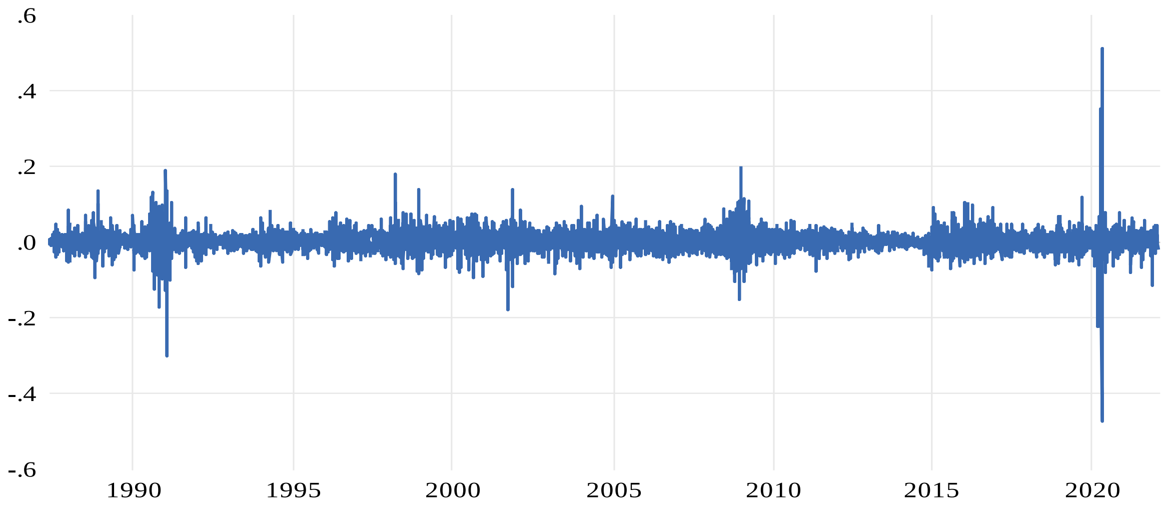
Before using GARCH models, it is extremely important to achieve valid investigations and statistical tests on the oil return series (Guo and Kilesen, 2005; Wang *et al.*, 2016; Charles and Darne, 2017; Yildirim, 2017). The analysed data series is represented by the determination of the daily Brent Crude Oil returns during 20 May 1987 to 24 January

2022. Information on Brent Crude Oil Prices (USD per Barrel) was found in the U.S. Energy Information Administration (EIA) database.

Daily oil returns were determined using the formula $\frac{P_t - P_{t-1}}{P_{t-1}}$ **(1)**

where p_t and p_{t-1} are the daily prices of Brent Crude Oil and t measures the time period (in days). In total, 8805 oil returns were calculated and their evolution can be seen in Figure 1.

Figure 1. The plot of the Brent Crude Oil Return Series in 1987-2022



Source: Authors' own research

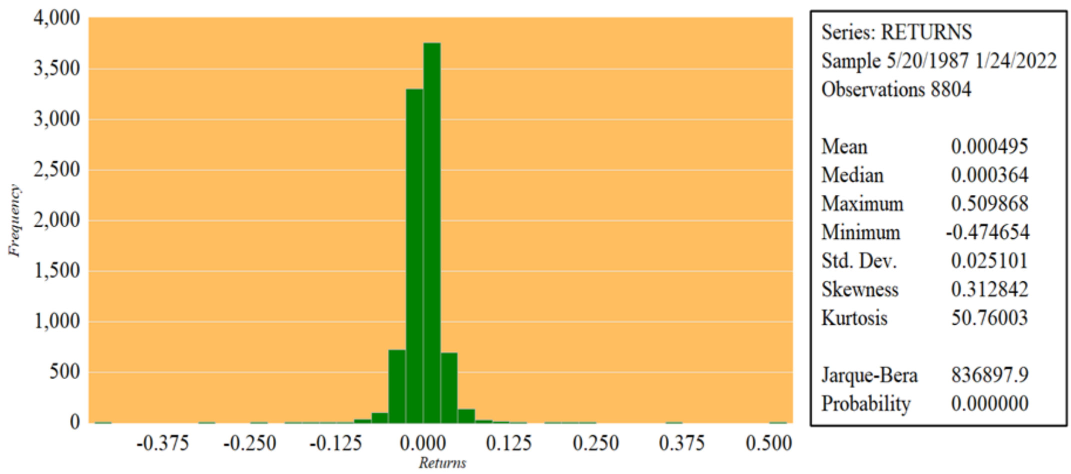
Figure 1 empirically shows the increasingly extreme values of daily oil returns, which continue to persist and amplify nowadays. For example, the onset of the Covid-19 Pandemic is a relevant factor for the persistence of the highest volatile level of Brent Crude Oil returns. In the first half of in 2020, these returns had negative values even up to -60%, a constant situation so far (negative values in the range of -20% and -40%).

Table 1. Descriptive statistics of the Brent Crude Oil Return Series

Mean	0,0495%
Standard Error	0,0268%
Median	0,0364%
Standard Deviation	2,5101%
Sample Variance	0,0630%
Kurtosis	47,78784549
Skewness	0,312895456
Minimum	-47,4654%
Maximum	50,99%
Count	8804

Source: Authors' own research

Figure 2. Histogram of the Brent Crude Oil returns



Source: Authors' own research

Table 1 and **Figure 2** show the main elements of statistical analysis of the Brent Crude Oil return series. Thus, in the analysed period, it is observed that the average return is approximately 0.05%, an extreme value close to 0%, and the standard deviation is around to 2.5%. The maximum value of oil returns is 51%, the minimum value is reached at -48%, which highlights the fluctuating variation in return series over important periods of time: the Global Financial Crisis (2008-2009); the effects of this crisis (2009-2019), respectively the COVID-19 Pandemic (2020-present).

Regarding the distribution of return series (**Figure 2**), the positive values of the Skewness (0.31) and Kurtosis (50.76) states highly present stylized facto in the analysis of financial return series, namely the fat-tail property of distribution or the presence of a leptokurtic and asymmetric to the right distribution. At the same time, based on the histogram, the oil returns do not match and do not tend towards a normal or Gaussian distribution N (0,1) and this fact is visible from the extremely high value of the Jarque-Bera test.

The next step was to identify the presence of ARCH terms, thus testing the level of increased probability of ARCH effects (q). This plays an important role in the use of ARCH-GARCH models and the manner in which the analysed time series can be estimated and fitted by these models (Yi *et al.*, 2021; Sekati *et al.*, 2020; Oyuna and Yasbin, 2021).

Table 2. The results for the presence of ARCH (q) effects

Variable	Coefficient	Std.error	Prob.
C	0.000358	0.00000433	0.0000
u_{t-1}^2	0.431906	0.009614	0.0000
Obs.×R-squared		1641.951 (Prob.Chi-Square =0.0000)	
F-statistic		218.027 (Prob.F =0.0000)	

Source: Authors' own research

Table 2 shows the results of the ARCH-LM test after running a linear regression having the lagged squared error term (u_{t-1}^2) as dependent variable.$(T-Q)\times R^2$ is 1641.951 and implicitly the associated p-value, which is lower than the 5% confidence level indicates that the oil return series, shows the existence of ARCH (q) effects and is recommended to use ARCH-GARCH models in estimating conditional volatility.

This fact is also observed at the p-value of lagged squared error terms (0.000) that is less than 5% confidence level and indicates the same type of conclusion: the presence of ARCH effects in the oil return series.

At the same time, the Augmented Dickey-Fuller (Dickey and Fuller, 1979) and Phillips-Perron (Phillips and Perron, 1988) tests were done to check for stationary of the oil return series. The results obtained can be seen in **Table 3**. In our case, ADF and PP have the statistical test values approximately equal to -93.00 and the associated p-value at 0.0001. As the test values are lower than the critical values by choosing the 1% confidence level, it can be certainly confirmed that the null hypothesis is rejected and the Brent Crude Oil returns series is stationary.

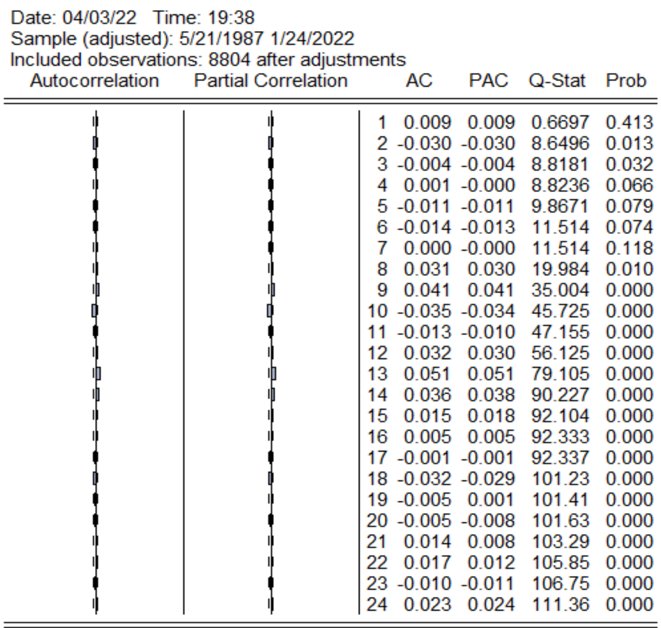
Table 3. Results of Stationary Tests

Augmented Dickey-Fuller (ADF)	**t-Statistic**	**Prob.**	**Test critical values (1% level)**
None	-92.96483*	0.0001	-2.565225
Constant	-92.99548*	0.0001	-3.430917
Constant+Linear Trend	-92.99190*	0.0001	-3.959078
Phillips-Perron (PP)	**t-Statistic**	**Prob.**	**Test critical values (1% level)**
None	-92.96483 *	0.0001	-2.565225
Constant	-92.99548*	0.0001	-3.430917
Constant+Linear Trend	-92.99190*	0.0001	-3.959078

*** =Statistically Significant at 1% Level**
Source: Authors' own research

At the same time, the impact and influence of the seasonal factors (i.e., seasonality) was tested on the oil return series. In this regard, we used the correlogram by testing the seasonality in our time series.

Figure 3. The correlogram of oil return series (at 24 lags)


Date: 04/03/22 Time: 19:38
Sample (adjusted): 5/21/1987 1/24/2022
Included observations: 8804 after adjustments

Lag	AC	PAC	Q-Stat	Prob
1	0.009	0.009	0.6697	0.413
2	-0.030	-0.030	8.6496	0.013
3	-0.004	-0.004	8.8181	0.032
4	0.001	-0.000	8.8236	0.066
5	-0.011	-0.011	9.8671	0.079
6	-0.014	-0.013	11.514	0.074
7	0.000	-0.000	11.514	0.118
8	0.031	0.030	19.984	0.010
9	0.041	0.041	35.004	0.000
10	-0.035	-0.034	45.725	0.000
11	-0.013	-0.010	47.155	0.000
12	0.032	0.030	56.125	0.000
13	0.051	0.051	79.105	0.000
14	0.036	0.038	90.227	0.000
15	0.015	0.018	92.104	0.000
16	0.005	0.005	92.333	0.000
17	-0.001	-0.001	92.337	0.000
18	-0.032	-0.029	101.23	0.000
19	-0.005	0.001	101.41	0.000
20	-0.005	-0.008	101.63	0.000
21	0.014	0.008	103.29	0.000
22	0.017	0.012	105.85	0.000
23	-0.010	-0.011	106.75	0.000
24	0.023	0.024	111.36	0.000

Source: Authors' own research

According to the results shown in **Figure 3**, it is observed that the oil return series follows an approximately linear pattern, with no oscillating and fluctuating movements and without the presence of sharply changes (i.e., up and downs, highs and lows changes). For these reasons, it is found that there is no seasonality in the analysed oil return series.

This part of investigating the Brent Crude Oil return series represents a special place in quantitative research, because useful information could be found by/for testing the stationary hypothesis, heteroskedasticity effect and by the descriptive statistical information about the average, variance, asymmetry indicators and the type of distributions, before applying the desired models of estimating and predicting the conditional volatility of these returns. From this point of view, our study was aligned with the preferred tools of analysis the oil return series used by Yildirim (2017); Zhang *et al.* (2019); Haque and Shaik (2021) or Mohammadi and Su (2010).

3.2. GARCH Models

In this section, we have applied different symmetrical and asymmetrical GARCH models to the view to obtain as accurate an estimate as possible of the crude oil volatility in our sample. Thus, for the beginning we applied the GARCH model (p, q), considering two equations: for the conditional mean, and for the conditional variance.

GARCH is a statistical modelling technique used to help predict the volatility of returns on financial assets and is appropriate for time series data where the variance of the error term is serially auto correlated following an autoregressive moving average process. For example, financial institutions typically use this model to estimate the volatility of returns for stocks, bonds, and market indices. The GARCH model (p, q) proposed by Bollerslev (1986) and extremely used in the financial econometric analysis has the following specification according to the formula below.

$$\text{GARCH (p,q)} \rightarrow h_t = \varphi + \sum_{k=1}^{p} \theta_k \times h_{t-k} + \sum_{i=1}^{q} b_i \times u_{t-i}^2 \qquad \textbf{(2)}$$

where: h_t = the conditional variance depends both on the past values of the shocks captured by the lagged squared error term and the past values of itself; u_{t-i}^2= the lagged squared error term; h_{t-i}= past values of itself; p= lagged terms of squared residual error term; q= lagged conditional variances; θ_k =GARCH coefficient and b_i =ARCH coefficient.

Specifically, by adapting the formula of the conditional variance of the GARCH (1,1) it is obtained this:

$$\text{GARCH (1,1)} \rightarrow h_t = \varphi + \theta_1 \times h_{t-1} + b_1 \times u_{t-1}^2 \qquad \textbf{(3)}$$

Being a model that can describe the way an economic agent tries to forecast volatility for the next period based on the long-term variance, previous variance (GARCH term) and volatility information previously observed (ARCH term), GARCH (1,1) aims at the simultaneous fulfilment of conditions below:

$$\theta_k; b_i > 0 \qquad \textbf{(4)}$$

$$\theta_k + b_i < 1 \qquad \textbf{(5)}$$

Also, to measure the crude oil volatility as realistically as possible (Yi *et al.*, 2021; Brandt and Gao, 2019; Wei *et al.*, 2017; Agnolucci, 2009; Ramzan *et al.*, 2012), we subsequently applied the extended GARRCH type models, respectively GARCH-in-Mean (GARCH-M (1,1) and Exponential GARCH (EGARCH).

The GARCH-M (p, q) model was introduced by Engle *et al.* (1987) and is obtained by entering the conditional variance or standard deviation (h_t or $\sqrt{h_t}$) into the mean return equation.

Specifically, financial theory suggests that an asset with a high volatility, on average, will have a higher return, and its effect is quantified by the coefficient of h_t from the mean equation (Codirlașu and Moinescu, 2010). Risk-averse investors may require a premium as a compensation to hold a risky asset. That premium is clearly a positive function of the risk (i.e. the higher the risk, the higher the premium should be). If the risk is captured by the conditional volatility, then the conditional variance may enter the conditional mean function of Y_t. Also, GARCH model allow the conditional mean to depend on its own conditional variance. It models a time-varying risk premium to explain asset returns.

That is, $Y_t = c + \alpha \times Y_{t-1} + \varepsilon \times h_t$ **(6)**

where c is the intercept; α, ε are the coefficients and Y_{t-1} represents the lagged value of the Brent Crude Oil return.

The GARCH-M (p,q) model is stated as:

$$h_t = \varphi + \sum_{k=1}^{p} \theta_k \times h_{t-k} + \sum_{i=1}^{q} b_i \times u_{t-1}^2 \quad \textbf{(7)}$$

It has often been observed in financial markets that economic agents understand volatility in a different ways and meanings depending on the sign of the daily change in the price of that financial asset.

Because the Brent Crude Oil benchmark price has been fluctuating quite a bit lately and it is reacting immediately to any kind of information, news, event, the next applied model was EGARCH (p, q), considered a non-linear or asymmetric model that can provide the most relevant prediction of the persistence of volatility over the time (Er and Fidan, 2018; Yildirim, 2017; Oyuna and Yaobin, 2021).

The EGARCH (p,q) model developed by Nelson (1991) is to capture the leverage effects on shocks (i.e. policies, information, news, events or incidents) on the market. It allows for the testing of asymmetries. With good (bad) news, asset (in our case, oil returns) tend to enter a state of tranquillity or turbulence and volatility decreases (increases). To do this, the log of the variance series is used.

The conditional variance for the EGARCH (p,q) model is specified as:

$$\log(h_t) = \varphi + \sum_{i=1}^{q} \eta_i \times \left|\frac{u_{t-i}}{\sqrt{h_{t-i}}}\right| + \sum_{i=1}^{q} \lambda_i \times \frac{u_{t-i}}{\sqrt{h_{t-i}}} + \sum_{k=1}^{p} \theta_k \times \log(h_{t-k}) \quad \textbf{(8)}$$

where φ = constant; η = ARCH effects; λ = asymmetric effects and θ= GARCH effects. Also, the EGARCH (1,1) is represented as:

$$\log(h_t) = \varphi + \eta_1 \times \left|\frac{u_{t-1}}{\sqrt{h_{t-1}}}\right| + \lambda_1 \times \frac{u_{t-1}}{\sqrt{h_{t-1}}} + \theta_1 \times \log(h_{t-1}) \quad \textbf{(9)}$$

Among the main advantages of using GARCH autoregressive models, we can certainly mention that these models outperform the better conditional volatility (i.e., high statistical significance) of the specified returns of financial and non-financial assets compared to a common OLS Regression or the ARCH model.

Another advantage of GARCH models is given by determining the three parameters that allow for an infinite number of squared roots to influence the conditional variance. Moreover, in general, for the normal period (pre and post-crisis), symmetric GARCH models**;** GARCH (1,1) and GARCH-M(1,1) perform better than the asymmetric GARCH.

But for fluctuation period (crisis period), asymmetric GARCH; EGARCH(1,1) model is preferred and commonly used.

On the other hand, multivariate GARCH models are the basic tools used to forecast correlations and covariance. For instance, time varying correlations are often estimated with Multivariate GARCH models that are linear in squares and cross products of the data.

According to Baybogan (2013), GARCH models provide improved estimations of the local variance (volatility), because these can be integrated into ARMA models being useful in modelling financial time series The main advantages of EGARCH models is clearly highlighted in the literature (Baybogan, 2013; Saltik *et al.*, 2016; Mohammadi, 2010). The strengths of this model are the logarithmic form which does not allow the positive constraint among parameters; EGARCH incorporates asymmetries in the change of volatility of returns. Also, this model can successfully define the change of volatility. Moreover, EGARCH model denotes that the conditional variance is an exponential function of the examined variables which ensures a positive character.

Regarding the main disadvantages or limitations in the use of autoregressive GARCH models, the same author (Baybogan, 2013) confirmed their inappropriateness in the cases where an asymmetric effect is usually observed and is registered from a different instability in the case of good and bad news. In the asymmetric models, upward and downward trends of returns are interpreted as bad and good news.

After estimating the three models (GARCH, GARCH-M and EGARCH), we continued to the diagnostic analysis for verifying the highest level of validity and explanatory power of these three used models.

This step involved checking the statistical significance of the resulted ARCH and GARCH parameters as well as performing the residuals tests such as: no heteroskedasticity, no autocorrelations in residuals series and the highest value of optimization function (Log Likelihood). In this sense, the applied tests were ARCH-LM Test, Lowest Schwarz Information Criteria and Durbin-Waston Significance Test. Depending on verifying these conditions, it was possible to make a ranking and consequently choosing the most accurate and fitted model in estimating the conditional variance equation. This aspect is found in studies conducted by Yildirim (2017); Sheng *et al.* (2020); Hu *et al.*, (2020), and Wei *et al.* (2010).

The last step was to build a conditional variance forecast for the entire sample (20 May 1987-24 January 2022) and for the modified sample (20 January 2020-24 January 2022). This period is considered an extremely uncertain period with multiple negative effects on the global crude oil price caused by the current COVID-19 Pandemic.

In a comparative manner, we were able to choose the most suitable forecasting model with the lowest Schwarz Information Criterion, lowest Root on Mean Square Error (RMSE), lowest Mean Absolute Error (MAE) and lowest Mean Absolute Percent Error (MAPE) for the two analysed periods (Bera and Higgins, 1993; Charles and Darne, 2017).

Finally, based on the estimated and predicted conditional variance equation, we show the estimated crude oil volatility by using the representative graphs created in EViews12. The entire methodological approach and the results obtained from the estimation of the GARCH (1,1), GARCH-M (1,1) and EGARCH (1,1) models were developed using the econometric software EViews12.

4. Results

In this section, we present and discuss the main results obtained from the analysis of the crude oil volatility by applying the autoregressive ARCH-GARCH models proposed by Bollerslev (1986), Engle and Bollerslev (1986), Nelson (1991) and used extensively in the researches of macroeconomic level (Yi *et al.*, 2021), energy and geopolitical sectors (Sheng *et al.*, 2020; Baker *et al.*, 2020; Sekati *et al.*, 2020) and implicitly in the financial and investment areas (Agnolucci, 2009; Wakeford, 2008; Zhang *et al.*, 2015; Cordirlașu and Moinescu, 2010).

From this perspective, we organize the results in a structured and staged manner, as follows: (i) presenting the results and discussing them after estimating the mean equation and the conditional variation for each model, respectively GARCH (1,1); GARCH-M (1.1) and EGARCH (1.1); (ii) investigating and diagnosing the applied autoregressive models and choosing the most suitable/reliable model in order to estimate the conditional variance; (iii) discussion of the main implications obtained from the forecast crude oil volatility analysis in the two time intervals/periods: 20 May 1987 - 24 January 2022 and 20 January 2020 - 24 January 2022.

4.1.The main results about the estimated conditional variance by using the GARCH models

Table 4 reflects the results obtained by applying the symmetric and linear GARCH (1,1) model.

Table 4. The results from GARCH (1,1) Model

Variables/Parameters	Coefficients	Std. error	Prob.
φ	0.00000052*	0.0000000587	0.0000
b_1	0.089008*	0.003464	0.0000
θ_1	0.905758*	0.003658	0.0000
c	0.000623*	0.000185	0.0007
α	0.036448*	0.011454	0.0015
The mean equation	$\widehat{Y_t}$= 0.000623 + 0.036448 × $\widehat{Y_{t-1}}$		
The variance equation	$\widehat{h_t}$=0.000000521+0.905758× $\widehat{h_{t-1}}$+ 0.089008× $\widehat{u_{t-1}}^2$		
Log likelihood	21801.18		
Durbin-Waston stat	2.053007		
Akaike information criterion (AIC)	-4.951989		
Schwarz criterion (SC)	-4.947966		
Hannan-Quinn criterion (HQ)	-4.950618		

*** =Statistically Significant at 1% Level**
Source: Authors' own research

We draw attention to the two resulting equations (the equation and the variance equation) and especially the compliance with the several conditions imposed by this model (Bollerslev, 1986; Bera and Higgins, 1993; Zakoian, 1994).

By visualizing the mean equation, it can be suggested that the coefficients are positive and highly statistically significant (1% p-value) and the average oil return is 0.000623 and its past value significantly predicts the current series by 0.03650. On the other hand, by analysing the conditional variance equation (h_t) can be confirmed that the coefficient of the constant variance term (φ), the ARCH term (b_1) and the GARCH parameter (θ_1) are positive, subunit and statistically significant at the 1% level of p-value.

This gives the results of the GARCH (1,1) model. The time-varying volatility includes a constant (φ =0.000000521) plus its past value (θ_1 =0.9058) and a component which depends on past errors (b_1 =0.089). Note that it took 14 iterations in EViews to reach the convergence ($Log\ likelihood = 21801$) and all the coefficients of the estimated conditional variance ($\widehat{h_t}$) specification meet the stability conditions: $0 < b_1 < 1$; $0 < \theta_1 < 1$ and $b_1 + \theta_1 < 1$.

These findings clearly establish the presence of time-varying conditional volatility of oil returns. Also, these results indicate that the persistence of volatility shocks, as represented by the sum of the ARCH and GARCH coefficients is large (the value is 0.9890) and this evidence is widely shown in recent studies (Brandt and Gao, 2019; Miao *et al.*, 2017; Yildirim, 2017). Therefore, it denotes that the effect of today's shocks remains in forecasts of variance for many periods in the future and studying the Brent Crude Oil volatility continues to be a relevant and interesting topic/subject in recent empirical studies (Baker *et al.*, 2016; Oyuna and Yaobin, 2021).

Regarding the results obtained by applying the GARCH-M (1,1) model, the first thing to notice indicates the variance term "GARCH" (which value is 1.4135) is statistically significant (p-value is less than 5% level) in the mean equation and its inclusion increases the significance of the GARCH coefficient (θ_1 =0.9065) in the variance equation. Also, the parameters are positive, and all are statistically significant at the 1% p-value and that means the conditions of GARCH-M (1,1) model are satisfied. The results obtained for this model are specified in **Table 5**.

Table 5. The results from GARCH-M (1,1) Model

Variables/Parameters	**Coefficients**	**Std. error**	**Prob.**
φ	0.000000514*	0.0000000585	0.0000
b_1	0.088279*	0.003438	0.0000
θ_1	0.906529*	0.003650	0.0000
c	0.000156	0.000261	0.5503
ε	1.413541**	0.553109	0.0106
α	0.035225*	0.011515	0.0022
The mean equation	$\widehat{Y_t}$= 0.000156 + 0.035225× $\widehat{Y_{t-1}}$ +1.413541 × $\widehat{h_t}$		
The variance equation	$\widehat{h_t}$=0.000000514+0.906529× $\widehat{h_{t-1}}$+0.088279× $\widehat{u_{t-1}}^2$		
Log likelihood	21805.55		
Durbin-Waston stat	2.060899		
Akaike information criterion (AIC)	-4.952754		
Schwarz criterion (SC)	-4.947926		
Hannan-Quinn criterion (HQ)	-4.951109		

*** =Statistically Significant at 1% Level ; ** Statistically Significant at 5% Level**

Source: Authors' own research

The novelty of the GARCH-M (1,1) model is that it captures the volatility (in the form of conditional variance or conditional standard deviation) in determining the average Brent Crude Oil return. Therefore, similar to other studies (Yildirim, 2017; Kutu and Ngalawa, 2017; Neshat *et al.*, 2018), the GARCH-M (1,1) model is able to illustrate the impact of investors' risk aversion in the face of fluctuating global oil price movements. On this line, our study also showed that the risk premium is significant to hedge against holding a risky asset (in our case, Brent Crude Oil return series). **Table 6** illustrates the specific results of the asymmetric EGARCH (1,1) model.

Table 6. The results from EGARCH (1,1) Model

Variables/Parameters	**Coefficients**	**Std.error**	**Prob.**
φ	-0.255227*	0.012877	0.0000
η_1	0.184823*	0.005764	0.0000
λ_1	-0.040188*	0.003510	0.0000
θ_1	0.984969*	0.001375	0.0000
c	0.000306***	0.000180	0.0885
α	0.040784*	0.010775	0.0002*
The mean equation	$\widehat{Y_t}$= 0.000306 + 0.040784× $\widehat{Y_{t-1}}$		
The variance equation	$\log(\widehat{h_t}) = -0.2552 + 0.184823 \times \left\lvert\frac{\widehat{u_{t-1}}}{\sqrt{h_{t-1}}}\right\rvert - 0.040188 \times \frac{\widehat{u_{t-1}}}{\sqrt{h_{t-1}}} + 0.984969 \times \log\widehat{(h_{t-1})}$		
Log likelihood	21827.82		
Durbin-Waston stat	2.061366		
Akaike information criterion (AIC)	-4.957813		
Schwarz criterion(SC)	-4.952986		
Hannan-Quinn criterion (HQ)	-4.956169		

*** Statistically Significant at 1% Level ; *** Statistically Significant at 10% Level**
Source: Authors' own research

Its popularity makes it used to estimate the risk associated with any type of financial asset or non-financial due, to the ability to analyse the asymmetric response to various shocks on the market. Thus, according to the mean equation, the coefficients are positive and are statistically at 1% level, respectively 5% level of p-value. At the same time, the obtained parameters at the level of conditional variation are statistically significant and confirms the stability of the GARCH Exponential (1,1) model (Saltik *et al.*, 2016; Mohammadi and Su, 2010; Herrera *et al.*, 2018).

The coefficient of the asymmetric term ($\lambda_1 = -0.04019$) is negative and this is statistically significant at the 1% level and in exponential terms ($e^{-0.040188}$) indicates that for the oil returns bad news has larger effect on the estimated volatility by 96%. Also, the GARCH parameter takes 0.9850 and is statistically significant at most restrictive probability level of 1% or 0.01. With the EGARCH (1,1) model, it was possible to calculate the total leverage effect which has the value of 4.25%, meaning investors can get a much larger exposure to the market by holding the Brent Crude Oil.

4.2.The investigation analysis of the applied GARCH Models

Starting from the specific hypotheses in using the ARCH-GARCH methods, literature (Yi *et al.*, 2021; Yildirim, 2017; Er and Fidan, 2013; Kulikova and Taylor, 2013; Zhang and Wang, 2015; Aye *et al.*, 2014) proposes various conditions that need to be fulfilled by each model applied. In this regard, the models should have least number of parameters, significant ARCH and GARCH parameters, high Log Likelihood ratio, lowest Schwarz Information Criteria, and also no heteroskedasticity and no autocorrelation in the residual or errors terms.

From this premise, we focused to the investigation and diagnosis of the three models used GARCH (1,1); GARCH-M (1,1) and EGARCH (1,1), the specific objective being to choose the most suitable model for estimating the conditional variance, respectively the model with the most criteria fulfilled/done. **Table 7** shows the results of the application of no heteroskedasticity and no autocorrelation test, as: ARCH-LM Test and Durbin-Waston Test.

Table 7. The results of evaluation models (Test for Residuals)

MODEL	ARCH-LM TEST	DURBIN-WASTON TEST
GARCH(1,1)	30.70352 (0.7184*)	1.999923
GARCH-M (1,1)	29.48024 (0.7705*)	1.999917
EGARCH (1,1)	39.77793 (0.3055*)	1.999983

***= the associated probability of ARCH-LM Test**
Source: Authors' own research

We can certainly say that the three models used (GARCH, EGARCH and GARCH-M) have successfully passed the residual conditions in the error series. Thus, the probabilities associated with the ARCH-LM test are above the 5% (0.05) level that the presence of ARCH (q) effects in the residual series is rejected and the errors are evenly distributed and homogeneous, validating the hypothesis of homoskedasticity.

The values, approximately equal to 2.00 of the Durbin-Waston Test confirmed the non-existence of the autocorrelation or serial correlations on the residual series. At the same time, according to **Table 8** and **Figure 4**, it is observed that the most accurate model in determining the conditional variance of Brent Crude Oil return series is the asymmetric EGARCH (1,1) model.

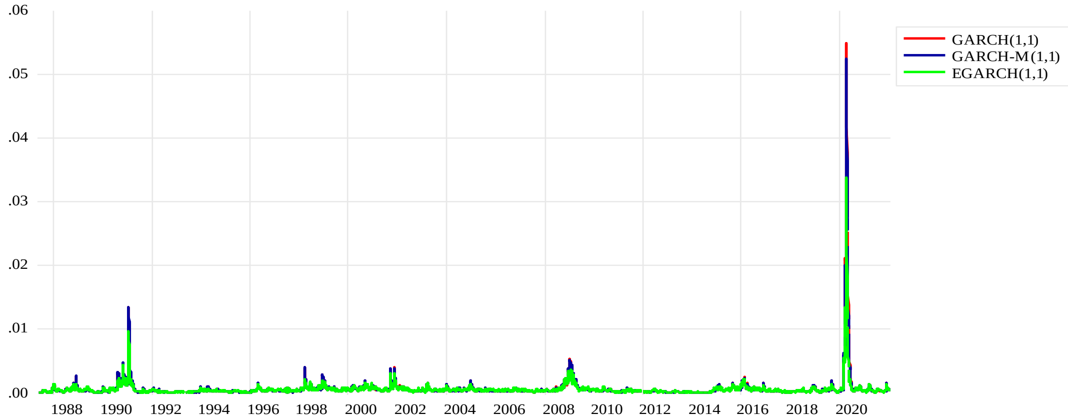
Compared to GARCH (1.1) and GARCH-M (1.1) models, this model is fully compliant and faithfully respect the main restrictive conditions imposed by the literature. In a similar approach are positioned the studies conducted by Yildirim (2017); Yang *et al.*, (2020), Saltik *et al.* (2016), where crude oil volatility is mainly measured through asymmetrical and non-parametric GARCH models in various periods of time.

Table 8. The best choosing estimated model

MODEL	ARCH significant?	GARCH significant?	LOG LIKELIHOOD	SCHWARTZ IC
GARCH (1,1)	Yes	Yes	21801.18	-4.947966
GARCH-M (1,1)	Yes	Yes	21805.55	-4.947926
EGARCH (1,1)	Yes	Yes	21827.82	-4.952986

Source: Authors' own research

Figure 4. The estimated conditional variance ($\widehat{h_t}$)



Source: Authors' own research

4.3. Forecast the Crude Oil Conditional Variance

After going through the diagnostic analysis of GARCH models, we performed a prediction analysis and forecasting the future conditional volatility of Brent Crude Oil return series.

According to the literature reviewed (Demirer *et al.*, 2018; Escribano and Valdes, 2017; Ahmed and Shabri, 2014), to obtain the best forecasting model it is essential to use the statistics from the each volatility model with the lowest values of Root Mean Square Error – a measure of how spread out these residuals are (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percent Error (MAPE) – represent the average of the absolute difference between the actual and predicted values in the dataset and there are the adequate techniques in measurement how the forecasting model is mostly accurate.

The analysed periods were 1987-2022, as well as 2020-2022. Choosing the last 2 years (2020-2022) is justified by the increased desire to observe the persistence and fluctuating and oscillating movement of Brent Crude Oil volatility in an increasingly uncertain, difficult and stressed period caused by the COVID-19 Pandemic.

Table 9. The Forecast Results (20 May 1987-24 January 2022)

MODEL	RMSE	MAE	MAPE	Rank
GARCH (1,1)	0.025106	0.010518	181.8033	2
GARCH-M (1.1)	0.025011	0.016300	180.9165	1
EGARCH (1.1)	0.025106	0.016318	183.9699	2

Source: Authors' own research

The results in **Table 9** show that the GARCH-M (1,1) model is fitted to predict the future conditional volatility, having the lowest values according to RMSE and MAE indicators. Also, interesting is that the GARCH (1,1) model has the lowest value of MAE (0.010518), but EGARCH (1,1) has the same value of 0.025106 as GARCH according to the obtained value of RMSE indicator.

On the other hand, the GARCH-M (1,1) is becoming the best forecasting model closed to the period 2020-2022, having the lowest values according to the performance indicators used (RMSE, MAE and MAPE).

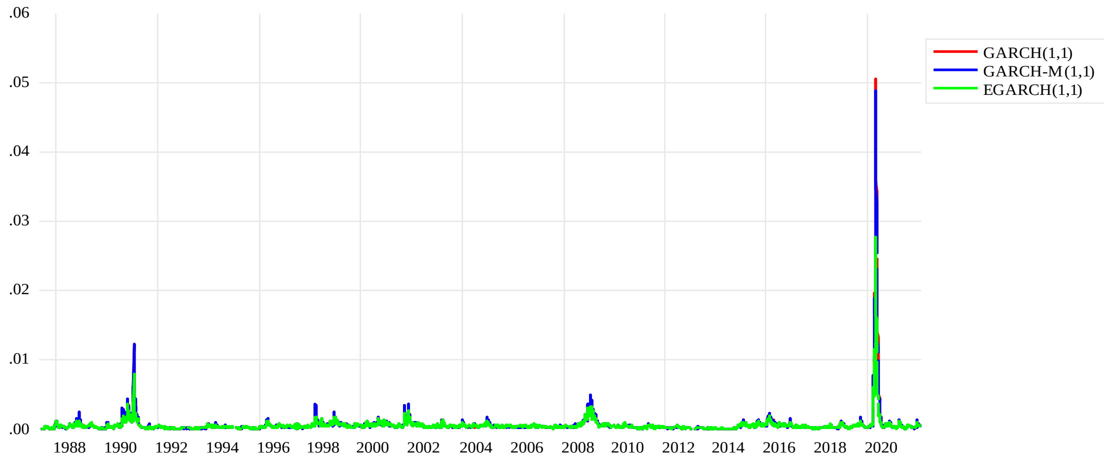
Table 10. The Forecast Results (20 January 2020-24 January 2022)

MODEL	RMSE	MAE	MAPE	Rank
GARCH (1,1)	0.050979	0.024694	153.9294	2
GARCH-M (1,1)	0.049623	0.024527	149.8150	1
EGARCH (1,1)	0.050965	0.024701	156.8613	3

Source: Authors' own research

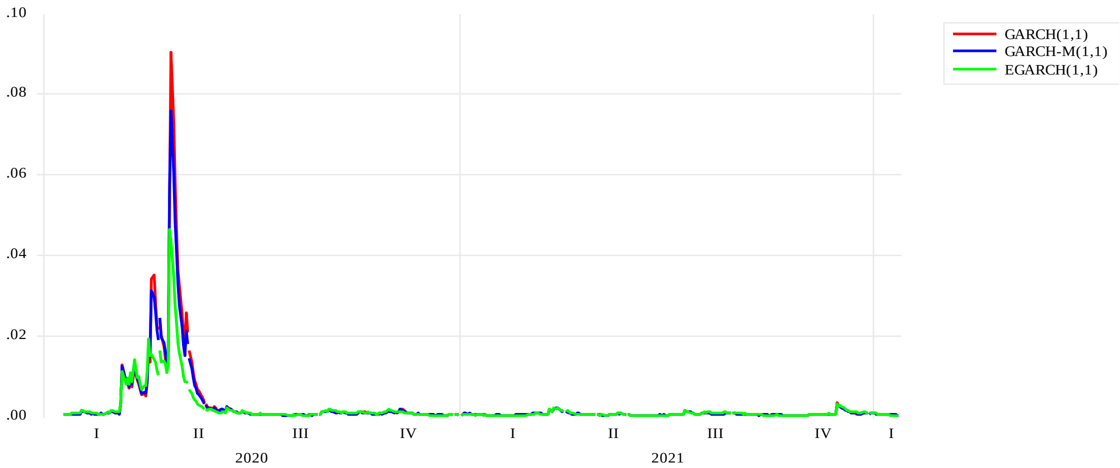
From this perspective, we can mention that our study differs from the preferred line in applying the asymmetric, nonlinear and exponential stochastic volatility models (i.e., TGARCH, PAGARCH, EGARCH, Heston model, etc.) in estimating and predicting the Brent Crude Oil variance (Oyuna and Yaobin, 2021; Bai *et al.*, 2020; Haque and Shaik, 2021). In conclusion, the oil return remains stable for the two periods, but indeed shows intense, high and persistent volatility which is indicated in **Figure 5** and **Figure 6**.

Figure 5. The forecast of crude oil volatility in 1987-2022



Source: Authors' own research

Figure 6. The forecast of crude oil volatility in 2020-2022



Source: Authors' own research

The evaluation of the accuracy of forecast models was performed on our autoregressive models applied, i.e. GARCH (1,1), GARCH- M (1,1) and EGACRH (1,1) for the entire sample - 27 May 1987-24 January 2022.

EViews allows you to use the comparison sample to construct a forecast evaluation statistic to provide a measure of forecast accuracy and perform Combination testing to determine whether a composite average of forecasts outperforms single forecasts. By using Eviews12 software program, we were able to run a series of accuracy tests in line with the testing and evaluating the forecast GARCH models (Diebold and Mariano, 1995; Hendry and Chong, 1986; Timmermann, 2006; Harvey *et al.*, 1998).

Our goal was to find a forecast that minimizes the errors. A number of measures are commonly used to determine the accuracy of a forecast. These include the mean absolute error (MAE), mean squared error (MSE) and root mean squared error (RMSE).

We used the Diebold-Mariano test (1995) to determine whether the two forecasts are significantly different. The Diebold-Mariano (DM) test was intended for comparing forecasts. The approach is to select the forecast that has the smaller error measurement based on one of the error measurements described in forecasting errors.

To determine whether one forecasting model predicts more accurately than another, we may test the equal accuracy hypothesis. The null hypothesis tells us that the both forecasts have the same accuracy. If this is rejected, we accept the alternative hypothesis: one forecast is better than the other. Since the DM statistics converge to a normal distribution, we can reject the null hypothesis at the 5% level if |DM| > 1.96 (Chen *et al.*, 2014). We compared the accuracy of these models, and the results are presented in the **Table 11** and **Table 12**.

Table 11. The results of Diebold-Mariano Test

Accuracy of Models	**GACH-M -> EGARCH**	**GARCH ->GARCH-M**	**GARCH ->EGARCH**
Abs Error	-14.75529 (0.0000*)	5.633996 (0.00000*)	-15.53027 (0.0000*)
Sq Error	-4.935658 (0.0000*)	4.514053 (0.0000*)	-4.962343 (0.0000*)

***= the associated probability of DM Test**

Source: Authors' own research

Table 12. The evaluation statistics

GARCH-M -> EGARCH				
Statistics	RMSE	MAE	MAPE	SMAPE
GARCH-M	**0.00000820**	**0.000000855**	**0.450000**	**0.456433**
EGARCH	0.000857	0.000130	11.55083	11.90563
GARCH -> EGARCH				
Statistics	RMSE	MAE	MAPE	SMAPE
GARCH	**0.000005820**	**0.000000855**	**0.458001**	**0.456433**
EGARCH	0.000788	0.0000125	11.52575	11.85430
GARCH-M->GARCH				
Statistics	RMSE	MAE	MAPE	SMAPE
GARCH	0.000857	**0.0000130**	12.92115	11.90563
GARCH-M	**0.000788**	0.000125	**12.79546**	**11.85430**

Source: Authors' own research

According to the DM test based on the absolute-error loss and squared-error loss, and their values is high than 1.96, it is confirmed that the null hypothesis is rejected at the 5% level of significance, that means the observed differences are significant and the forecasting accuracy is different in each case. The evaluation statistics (**Table 12**) shows us the GARCH-M model is better than EGARCH, while GARCH model is less accurate than GARCH-M. Also, the GARCH model is more accurate than asymmetric EGARCH model.

Consequently, the analysis performed on estimation, valuation and forecasting the crude oil return in an accurate and precise quantitative by applying the Generalized Autoregressive Conditional Heteroskedasticity models is becoming necessary as well extremely important to inform the participants about the main characteristics in holding a risky asset or portfolio in international financial markets. The need for modelling and forecasting the volatility is because the investors are not only interested in the average returns of a stock or asset, but also in its risk and they need proper information to analyse the gains or losses from the erratic behaviour of financial or non-financial assets.

5. Conclusion

Identification, measurement and forecasting the diverse and multiple types of risk are the main elements of the modern field of finance, where more and more specialists and researchers are concerned with finding and using the most appropriate and accurate methods for estimating this volatility.

Currently, it is becoming more and more difficult to estimate risk-specific indicators over an extremely unpredictable and uncertain period filled of numerous events and facts that has increased volatility in international financial markets (i.e., terrorist events, financial crises, pandemics, cyber-attacks, etc.). Especially, commodities in terms of crude oil, silver, platinum, natural gas or gold are the mostly exposed to sharply increase and decrease in price (Yildirim, 2017). In this regard, the main objective of our study was to measure conditional variance at the Brent Crude Oil return series in the 1987-2022.

The quantitative approach was suggested by applying ARCH- GARCH autoregressive models GARCH proposed by Bollerslev (1986), Engle and Bollerslev (1986) and extended by Nelson (1991). ARCH and GARCH models have become important tools in the analysis of time series data, particularly in financial applications. These models are especially useful when the goal of the study is to analyse and forecast volatility.

It is justified that GARCH models describe financial markets in which volatility can change, becoming more volatile during periods of financial crises or world events and less volatile during periods of relative calm and steady economic growth (Engle and Bollerslev, 1986; Nelson 1991; Bera and Higgins, 1993).

Prior to the application of these models, a first step was focused on statistical analysis and descriptive of the daily oil return series during the mentioned period. Similar to other empirical studies (Dunn and Holloway, 2012; Charles and Darne, 2017; Yildirim, 2017; Wang *et al.*, 2016), the results obtained showed that the oil return series shows the ARCH (q) effects, is stationary at the first level of integration (were used ADF and PP tests), as well as the existence of a leptokurtic distribution and the identification of the property "fat tails" based on the construction of the histogram.

Analysing the daily oil returns, it was possible to observe the fluctuating trends and, implicitly the specified crude oil volatility. Also, the Brent Crude Oil returns have more and more extreme values, especially in the COVID-19 Pandemic (actual) period, attracting negative values up to -50%.

Results from GARCH models, GARCH (1.1), GARCH-M (1.1), EGARCH (1,1) on the estimation of conditional variance ($\widehat{h_t}$) indicated that the asymmetrical EGARCH model (1,1) is the most reliable and fitted model in achieving the objective in our study. At the same time, the GARCH coefficient (θ_1 = 0.9850) and the negative value of the asymmetric coefficient (λ_1 = - 0.040) are statistically significant at 1% level and suggest both that bad news has larger effect on the volatility than good news in the market for the analysed Brent Crude Oil return series.

On the other hand, each model complied with the restrictive conditions imposed, indicating statistical significance for each parameter (p-value = 1%), these being subunits and positive. In each case, the sum between ARCH term and GARCH term was less than 1, another restrictive condition being met, suggesting that Brent Crude Oil returns shocks are persistent and remain extremely high. For example, the values indicated by GARCH (1.1), GARCH-M (1.1) are 0.99, and in the case of EGARCH (1.1) is 0.96.

In order to determine the level of accuracy of the three models, the main conditions were tested in compliance with the homoscedasticity hypothesis and serial correlations regarding the error distribution. Therefore, the p-values are higher than 5% for the ARCH-LM test, indicating that each model (GARCH, EGARCH and GARCH-M) complies with the condition of "no heteroskedasticity" in residuals. In the similar way, the values of Durbin-Waston Test show that the residuals of oil returns do not have serial correlation or autocorrelation.

The contribution of our study consists in conducting a forecast analysis of conditional variance in two different periods, respectively: 20 May 1987-24 January 2022 and 20 January 2020-24 January 2022. Crude oil volatility forecasts are more and more important for making any macroeconomic decision, having financial implication in managing assets, options, derivates or portfolios. Although the last period is extremely imprecise, and the global oil price has recorded extraordinary situations (i.e. sharp drop in the demand for oil due to the spread of COVID-19 Pandemic or the negative oil prices in April 2020), more and more reviewed studies showed the asymmetric GARCH models are preferred in estimating and forecasting crude oil volatility.

A different thing noticed by our study highlights the ability of symmetric GARCH (1,1) and GARCH-M (1,1) models in forecasting the crude oil volatility in the last fluctuating period (2020-2022). The forecasting results showed that for the whole period, GARCH (1,1) is the model with the highest level of crude oil volatility forecast compared to the model GARCH-M (1,1) which is suitable for the period 2020-2022.

In order to choose the best forecasting GARCH model, we also conducted an assessing analysis using the lowest values of Schwarz Information Criterion, Root on Mean Square Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percent Error (MAPE). This evidence suggests that the present study opts for the use of symmetrical methods (GARCH-M) in the forecasting crude oil volatility compared to the results use of asymmetric models (T-GARCH, EGARCH, Heston model) in most empirical and quantitative studies (Yi *et al.*, 2021; Oyuna and Yaobin, 2021; Haque and Shaik, 2021; Wang *et al.*, 2016).

Consequently, by applying the autoregressive models it could be highlighted that the Brent Crude oil return series are still volatile in the near future. Also, since ARCH term and GARCH term is smaller than one, we can conclude that extremely high volatility will have serious implications for government policymakers and will continue to be a widespread and debated topic in the literature.

Moreover, the Brent Crude Oil return series exhibits volatility clustering; periods when large changes are followed by further large changes and periods when small changes are followed by further small changes.

Indeed, our study presents some limitations. A major limitation derives from the framework and the main conditions of autoregressive GARCH models used. These models assume deterministic volatility based on past returns and conditional variances, and most often ignores the determination of volatility based on intraday prices. For these reasons, a future research direction to be continued should refer to the alternative methods in estimating intraday volatility on oil return series.

Another limitation derivates from the extremely rigorous and in-depth theoretical framework of the estimation and forecast of the conditional variance of financial and non-financial assets. Volatility is perceived as something unpredictable and that changes extremely suddenly and constant. Thus, it is recommended a rigorous and constant documentation about this relevant topic in the field of risk management.

Taking into account that our study has a practical approach, a possible future direction in research should consist in making a comparative analysis on the estimation of variance over the other return series (i.e. natural gas, gold, silver, coal, etc). At the same time, real difficulties may arise in choosing the most suitable GARCH model. For these reasons other methods must be used, for example i.e. ARIMA process, OLS Regression or VAR approach. To solve these problems, considering the use of machine learning techniques could be a way to provide high significance level in estimating the variance.

References:

Adelman, A. (2000). Determinants of growth and development of the Australian economy. *Australian Journal of Economics, 14* (3), 19-42.

Agnolucci, P. (2009). Volatility in crude oil futures: A comparison of the predictive ability of GARCH and implied volatility models. *Energy Economics, 31* (2), 316-321.https://doi.org/10.1016/j.eneco.2008.11.001

Ahmed, R. A., & Shabri, A. B. (2014). Daily crude oil price forecasting model using ARIMA, Generalized Autoregressive conditional heteroscedastic and Support Vector Machines. *American Journal of Applied Sciences, 11* (3), 425-432. https://doi.org/10.3844/ajassp.2014.425.432

Aye, G. C., Dadam, V., Gupta, R., & Mamba, B. (2014). Oil price uncertainty and manufacturing production. *Energy Economics, 43*, 41-47.https://doi.org/10.1016/j.eneco.2014.02.004

Bai, L., Wei, Y., Wei, G., Li, X., & Zhang, S. (2020). Infectious disease pandemic and permanent volatility of international stock markets: A long-term perspective. *Finance Research Letters, 40*, 101709.https://doi.org/10.1016/j.frl.2020.101709

Baker, S. R., Bloon, N., & Davis, S. H. (2016). Measuring Economic Policy Uncertainty. *The Quarterly Journal of Economics, 131* (4), 1593-1636. https://doi.org/10.1093/qje/qjw024

Baker, S. R., Bloom, N., Davis, S. J., Kost, K., Sammon, M., & Viratyosin, T. (2020). The Unprecedented Stock Market Reaction to COVID-19. *The Review of Asset Pricing Studies, 10* (4), 742-758. https://doi.org/10.1093/rapstu/raaa008

Baybogan, B. (2013). Empirical Investigation of Mgarch Models. *Journal of Statistical and Econometric Methods, 2* (3), 1-7. https://ideas.repec.org/a/spt/stecon/v2y2013i3f2_3_7.html

Bera, A. K., & Higgins, M. L. (1993). ARCH Models: Properties, estimation and testing. *Journal of Economic Surveys, 7* (4), 305-366. https://doi.org/10.1111/j.1467-6419.1993.tb00170.x

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics, 31* (3), 307-327.https://doi.org/10.1016/0304-4076(86)90063-1

Brandt, M. W., & Gao, L. (2019). Macro fundamentals or geopolitical events? A textual analysis of news events for crude oil. *Journal of Empirical Finance, 51*, 64-94.https://doi.org/10.1016/j.jempfin.2019.01.007

Charles, A., & Darne, O. (2017). Forecasting crude-oil market volatility: Further evidence with jumps. *Energy Economics, 67*, 508-519.https://doi.org/10.1016/j.eneco.2017.09.002

Chen, H., Wan, Q., & Wang, Y. (2014). Refined Diebold-Mariano Test Methods for the Evaluation of Wind Power Forecasting Models. *Energies, 7*, 4185-4198. doi:10.3390/en7074185

Codirlașu, A., & Moinescu, B. (2010). *Econometrie avansată [Advanced econometrics].* https://www.researchgate.net/publication/326410312_Econometrie_avansata.

Demirer, R., Gupta, R., Suleman, T., & Wohar, M. E. (2018). Time-varying rare disaster risks, oil returns and volatility. *Energy Economics, 75*, 239-248.https://doi.org/10.1016/j.eneco.2018.08.021

Dickey, D. A., & Fuller, W. A. (1979). Distribution of the Estimators for Autoregressive Time Series With a Unit Root. *Journal of the American Statistical Association, 74* (366), 427-431.https://doi.org/10.2307/2286348

Diebold, F., & Mariano, R. (1995). Comparing Predictive Accuracy. *Journal of Business & Economic Statistics, 13* (3), 253-263. https://www.sas.upenn.edu/~fdiebold/papers/paper68/pa.dm.pdf

Dunn, S., & Holloway, J. (2012, September). The Pricing Of Crude Oil. *RBA Bulletin Reserve Bank of Australia* , pg. 65-74.https://www.rba.gov.au/publications/bulletin/2012/sep/pdf/bu-0912-8.pdf

Engle, R. F., & Bollerslev, T. (1986). Modelling the persistence of conditional variances. *Econometric Reviews, 5* (1), 1-50.https://doi.org/10.1080/07474938608800095

Engle, R., Lilien, D., & Robins, R. P. (1987). Estimating Time Varying Risk Premia in the Term Structure: The Arch-M Model. *Econometrica, 55*(2), 391-407.https://doi.org/10.2307/1913242

Er, Ș., & Fidan, N. (2013). Modelıng Istanbul Stock Exchange-100 Daıly Stock Returns: A Nonparametric GARCH A pproch. *Journal of Business, Economics & Finance, 2* (1), 36-50. https://dergipark.org.tr/tr/download/article-file/374601

Escribano, G., & Valdes, J. (2017). Oil Prices: Governance Failures and Geopolitical Consequences. *Geopolitics, 22* (3), 693-718.https://doi.org/10.1080/14650045.2016.1254621

Guo, H., & Kilesen, K. L. (2005, November/December). Oil Price Volatility and U.S. Macroeconomic Activity. *Federal Reserve Bank of St. Louis Review, 87* (6), pp. 669-683.https://files.stlouisfed.org/files/htdocs/publications/review/05/11/KliesenGuo.pdf

Haque, M. I., & Shaik, A. R. (2021). Predicting Crude Oil Prices During a Pandemic: A Comparison of Arima an Garh Models. *Montenegrin Journal of Economics, 17* (1), 197-207.

Harvey, D. I., Leybourne, S. J., & Newbold, P. (1998). Tests for Forecast Encompassing. *Journal of Business & Economic Statistics, 16* (2), 254-259. https://doi.org/10.2307/1392581

Hendry, D., & Chong, Y. Y. (1986). Econometric Evaluation of Linear Macro-Economic Models. *Review of Economic Studies, 53* (4), 671-690. https://doi.org/10.2307/2297611

Herrera, A. M., Hu, L., & Pastor, D. (2018). Forecasting crude oil price volatility. *International Journal of Forecasting, 34* (4), 622-635. https://doi.org/10.1016/j.ijforecast.2018.04.007

Hu, M., Zhang, D., Ji, Q., & Wei, L. (2020). Macro factors and the realized volatility of commodities: A dynamic network analysis. *Resources Policy, 68*, 101813.https://doi.org/10.1016/j.resourpol.2020.101813

Jin, G. (2008). The Impact of Oil Price Shock and Exchange Rate Volatility on Economic Growth: A Comparative Analysis for Russia, Japan, and China. *Research Journal of International Studies*, 8 (11), 98-111.

Kulikova, M., & Taylor, D. (2013). Stochastic volatility models for exchange rates and their estimation using quasi-maximum-likelihood methods: an application to the South African Rand. *Journal of Applied Statistics, 40* (3), 495-507.https://doi.org/10.1080/02664763.2012.740791

Kutu, A. A., & Ngalawa, H. (2017). Modelling Exchange Rate Volatility and Global Shocks in South Africa. *Acta Universitatis Danubius. Œconomica, 13* (3), 178-193. http://journals.univ-danubius.ro/index.php/oeconomica/article/view/3918/4167

Lipsky, J. (2009; 18 March). *Economic Shifts and Oil Price Volatility.* Internationalu Monetary Fund (IMF). Vienna: 4th OPEC International Seminar.https://www.imf.org/en/News/Articles/2015/09/28/04/53/sp031809

Liu, J., Ma, F., Tang, Y., & Zhang, Y. (2019). Geopolitical risk and oil volatility: A new insight. *Energy Economics, 84*, 104548.https://doi.org/10.1016/j.eneco.2019.104548

Manera, M., Longo, C., Markandya, A., & Scarpa, E. (2007). Evaluating the Empirical Performance of Alternative Econometric Models for Oil Price Forecasting. *FEEM Fondazione Eni Enrico Mattei Research Paper Series [Working Paper]* (4), 1-49.https://papers.ssrn.com/sol3/papers.cfm?abstract_id=958942#

Miao, H., Ramchander, S., Wang, T., & Yang, D. (2017). Influential factors in crude oil price forecasting. *Energy Economics, 68*, 77-88. https://doi.org/10.1016/j.eneco.2017.09.010

Mohammadi, H., & Su, L. (2010). International evidence on crude oil price dynamics: Applications of ARIMA-GARCH models. *Energy Economics, 32* (5), 1001-1008. https://doi.org/10.1016/j.eneco.2010.04.009

Narayan, P. K., & Narayan, S. (2007). Modelling oil price volatility. *Energy Policy, 35* (12), 6549-6553.https://doi.org/10.1016/j.enpol.2007.07.020

Nelson, D. B. (1991). Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica, 59* (2), 347-370.https://doi.org/10.2307/2938260

Neshat, N., Hadian, H., & Behzad, M. (2018). Nonlinear ARIMAX model for long –term sectoral demand forecasting. *Management Science Letters, 8* (6), 581-592. http://www.growingscience.com/msl/Vol8/msl_2018_45.pdf

Oyuna, D., & Yaobin, L. (2021). Forecasting the Crude Oil Prices Volatility With Stochastic Volatility Models. *SAGE Open* .https://doi.org/10.1177/21582440211026269

Pagan, A. R., & Schwert, W. G. (1990). Alternative models for conditional stock volatility. *Journal of Econometrics, 45* (1-2), 267-290.https://doi.org/10.1016/0304-4076(90)90101-X

Phillips, P. C., & Perron, P. (1988). Testing for a unit root in time series regression. *Biometrika, 75* (2), 335-346.https://doi.org/10.1093/biomet/75.2.335

Ramzan, S., Ramzam, S., & Zahid, F. M. (2012). Modeling and forecasting exchange rate dynamics in Pakistan using ARCH family of models. *Electronic Journal of Applied Statistical Analysis, 5* (1), 15-29.10.1285/i20705948v5n1p15

Saltik, O., Degirmen, S., & Ural, M. (2016). Volatility Modelling In Crude Oil and Natural Gas Prices. *Procedia Economics and Finance, 38*, 476-491. https://doi.org/10.1016/S2212-5671(16)30219-2

Sekati, B. N., Tsoku, J. T., & Metsileng, L. D. (2020). Modelling the oil price volatility and macroeconomic variables in South Africa using the symmetric and asymmetric GARCH models. *Cogent Economics & Finance, 8* (1), 1792153.https://doi.org/10.1080/23322039.2020.1792153

Sheng, X., Gupta, R., & Ji, Q. (2020). The impacts of structural oil shocks on macroeconomic uncertainty: Evidence from a large panel of 45 countries. *Energy Economics, 91*.https://doi.org/10.1016/j.eneco.2020.104940

Timmermann, A. (2006). Forecast Combinations. In *Handbook of Economic Forecasting* (Vol. 1, pp. 135-196). Elsevier. https://doi.org/10.1016/S1574-0706(05)01004-9

Wakeford, J. (2008). The Impact of Oil Price Shocks on the South African Macroeconomy: History and Prospects. *Accelerated and Shared Growth in South Africa: Determinants, Constraints and Opportunities; DPRU Forum*, Johannesburg.

Wang, Y., & Wu, C. (2012). Forecasting energy market volatility using GARCH models: Can multivariate models beat univariate models? *Energy Economics, 34* (6), 2167-2181.https://doi.org/10.1016/j.eneco.2012.03.010

Wang, Y., Wu, C., & Yang, L. (2016). Forecasting crude oil market volatility: A Markov switching multifractal volatility approach. *International Journal of Forecasting, 32* (1), 1-9.https://doi.org/10.1016/j.ijforecast.2015.02.006

Wei, Y., Liu, J., Lai, X., & Hu, Y. (2017). Which determinant is the most informative in forecasting crude oil market volatility: Fundamental, speculation, or uncertainty? *Energy Economics, 68*, 141-150.https://doi.org/10.1016/j.eneco.2017.09.016

Wei, Y., Wang, Y., & Huang, D. (2010). Forecasting crude oil market volatility: Further evidence using GARCH-class models. *Energy Economics, 32* (6), 1477-1484.https://doi.org/10.1016/j.eneco.2010.07.009

Yang, C., Lv, F., Fang, L., & Shang, X. (2020). The pricing efficiency of crude oil futures in the Shanghai International Exchange. *Finance Research Letters, 36,* 101329.https://doi.org/10.1016/j.frl.2019.101329

Yaziz, S. R., Ahmad, M. H., Nian, L. C., & Muhammad, N. (2011). A Comparative Study on Box-Jenkins and Garch Models in Forecasting Crude Oil Prices. *Journal of Applied Sciences, 11* (7), 1129-1135.10.3923/jas.2011.1129.1135

Yi, A., Yang, M., & Li, Y. (2021). Macroeconomic Uncertainty and Crude Oil Futures Volatility - Evidence from China Crude Oil Futures Market. *Frontiers in Environmental Science, 9*, 1-13.10.3389/fenvs.2021.636903

Yildirim, H. (2017). ARCH-GARCH Model on volatility of crude oil. *International Journal of Disciplines Economics and Administrative Sciences Studies (IDEAstudies), 3* (3), 17-22. http://www.ideastudies.com/DergiTamDetay.aspx?ID=11

Zakoian, J.-M. (1994). Threshold heteroskedastic models. *Journal of Economic Dynamics and Control, 18* (5), 931-955. https://doi.org/10.1016/0165-1889(94)90039-6

Zhang, J.-L., Zhang, Y.-J., & Zhang, L. (2015). A novel hybrid method for crude oil price forecasting. *Energy Economics, 49,* 649-659.https://doi.org/10.1016/j.eneco.2015.02.018

Zhang, Y.-J., & Ma, S.-J. (2021). Exploring the dynamic price discovery, risk transfer and spillover among INE, WTI and Brent crude oil futures markets: Evidence from the high-frequency data. *International Journal of Finance & Economics, 26* (2), 2414-2435.

Zhang, Y.-J., & Wang, J. (2015). Exploring the WTI crude oil price bubble process using the Markov regime switching model. *Physica A: Statistical Mechanics and its Applications, 421(C)*, 377-387. https://doi.org/10.1016/j.physa.2014.11.051

Zhang, Y.-J., Yao, T., He, L.-Y., & Ripple, R. (2019). Volatility forecasting of crude oil market: Can the regime switching GARCH model beat the single-regime GARCH models? *International Review of Economics & Finance, 59,* 302-317.https://doi.org/10.1016/j.iref.2018.09.006